Study of Near Consensus Complex Social Networks Using Eigen Theory

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Abstract

This paper extends the definition of an exact consensus complex social network to that of a near consensus complex social network. A near consensus complex social network is a social network with nontrivial topological features and steady state values of the decision certitudes of the majority of the nodes being either higher or lower than a threshold value. Based on this definition, the relationships among the vectors representing the steady state values of the decision certitudes of the nodes, the influence weight matrix of a near consensus complex social network and the set of vectors representing the initial state values of the decision certitudes of the nodes that satisfies a given near consensus specification are investigated. Finally, some practical examples of near consensus complex social networks are illustrated.

Keywords: Complex networks, social networks, consensus networks, exact consensus, near consensus.

1 Introduction

A complex network is a network with nontrivial topological features [1]. Complex networks appear in various contexts in our daily life. Over the last decade, complex networks have drawn attentions of many researchers from different fields of physical and social sciences, such as mathematics [2], computer science [3], engineering [4], [11], [12], biology [5], economics [6] and social sciences [7]. A social network is a network made of individuals, called nodes, which interact with each others [7]. As many social networks have nontrivial topological features, social networks are actually complex networks.

In the past, complex social networks were investigated using lattice and graph theory [8]. Existing studies are based on an exact consensus property (i.e., the steady state values of the decision certitudes of all the nodes are equal to either 1 or -1 [9]). In reality, the steady state values of the decision certitudes of the majority of the nodes are either higher or lower than a threshold value [10]. Thus, it is useful to extend the definition of the exact consensus property to that of a near consensus property. In this paper, the near consensus property is referred to the property that the steady state values of the decision certitudes of the majority of the nodes are either higher or lower than a threshold value. Deriving the near consensus property is a key to the analysis of practical social networks [10]. However, by extending the definition to the near consensus property, the relationships among the vectors representing the steady state values of the decision certitudes of the nodes, the influence weight matrix of a near consensus complex social network and the set of vectors representing the initial state values of the decision certitudes of the nodes that satisfies a given near consensus specification are unknown. The aim of this paper is to address the above issue.

Note that this investigation is challenging as the set of vectors representing the steady state values of the decision certitudes of the nodes that satisfies a given near consensus specification is nonconvex. So far, no results have been reported on the near consensus complex social networks.

2 Main Results

The necessary and sufficient condition for the vectors representing the decision certitudes of the nodes of complex social networks converging to a nonzero fixed vector for each nonzero initial condition as follows. (i) All the eigenvalues of the weight influence matrix are inside or on the circle; (ii) there exists some eigenvalues on the unit circle; and (iii) for those eigenvalues are on the unit circle, they are equal to one.

Suppose that the wight influence matrix is diagonalizable. Assume that there exists at least one eigenvectors of the wight influence matrix with all their elements equal to 1 or -1 and the corresponding eigenvalues equal to 1, then the sum of the elements in each row of the weight influence matrix is equal to one.

For a given nonzero initial state and a diagonalizable weight influence matrix, if the state vector converges to a nonzero vector, then the steady state vector can be expressed as a linear combination of the eigenvectors corresponding to the eigenvalues equal to one. The corresponding linear combination coefficients depend on the initial conditions. On the other hand, for a given steady state vector and a diagonalizable weight influence matrix, the initial state is not uniquely defined. In fact, the initial state is in the linear variety of the subpace spanned by the set of eigenvectors corresponding to the eigenvalues equal to one and translated by the steady state vector.

References

[1] D.J. Watts, and S.H. Strogatz. Collective dynamics of 'small-world networks'. *Nature*, 393:440–442, 1998.

[2] P. DeLellis, M. diBernardo, F. Garofalo, and D. Liuzza. Analysis and stability of consensus in networked control systems. *Applied Mathematics and Computation*, in press.

[3] J. Risson, and T. Moors. Survey of research towards robust peer-to-peer networks: search methods. *Computer Networks*, 50:3485–3521, 2006.

[4] W. Yu, J. Cao, G. Chen, J. Lu, J. Han, and W. Wei. Local synchronization of a complex network model. *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 39:230–241, 2009.

[5] P.T. Foteinou, E. Yang, and I.P. Androulakis. Networks, biology and systems engineering: a case study in inflammation. *Computers and Chemical Engineering*, 33:2028–2041, 2009.

[6] D. Acemoglu, A. Ozdaglar, and A. ParandehGheibi. Spread of (mis)information in social networks. *Games and Economic Behavior*, in press.

[7] S. Goldfinch. The old boys' network? Social ties and policy consensus amongst Australian and New Zealand economic policy elites. *Policy and Society*, 21:1–25, 2002.

[8] J. Yu, E. Noel, and K.W. Tang. A graph theoretic approach to ultrafast information distribution: Borel Cayley graph resizing algorithm. *Computer Communications*, in press.

[9] J. Zhou, and Q. Wang. Convergence speed in distributed consensus over dynamically switching networks. *Automatica*, 45:1455–1461, 2009.

[10] R.J. Johnston, C.J. Pattie, D.F.L. Dorling, I. MacAllister, H. Tunstall, and D.J. Rossiter. Social locations, spatial locations and voting at the 1997 British general election: evaluating the sources of Conservative support. *Political Geography*, 20:85–111, 2001.

[11] Z. Hou, L. Cheng, and M. Tan. Decentralized robust adaptive control for the multiagent system consensus problem using neural networks. *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 39:636–647, 2009.

[12] Y. Cao, and W. Ren. Optimal linear-consensus algorithms: an LQR perspective. *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 40:819–830, 2010.